

# Statistics

## Lecture 8



Feb 19-8:47 AM

Data {
 

- 1) Qualitative
- 2) Quantitative {
  - 1) Discrete
  - 2) Continuous

S&amp;14

Let  $x$  be a discrete random variable with prob. dist.  $P(x)$ .

It gives the prob. of all possible outcomes.

1) Table or chart

2) Graph

3) formula

4) use def. of Prob.

Oct 21-6:52 PM

Some rules:

$$1) 0 \leq P(x) \leq 1$$

$$2) \sum P(x) = 1$$

$$3) P(x) = 1 \longleftrightarrow \text{Sure event}$$

$$4) P(x) = 0 \longleftrightarrow \text{Impossible event}$$

$$5) 0 < P(x) \leq .05 \longleftrightarrow \text{Rare event}$$

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Consider the chart below

$x$	$P(x)$
1	.3
2	.5
3	.2

$$1) \text{ verify } \sum P(x) = 1 \checkmark$$

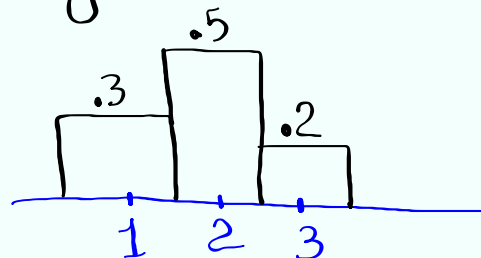
$$.3 + .5 + .2 = 1 \checkmark$$

$$2) P(x \leq 2) = .3 + .5 = \boxed{.8}$$

3) Draw Prob. dist. histogram

$x \rightarrow$  Midpoint

$P(x) \rightarrow$  Rel. F.



Oct 21-7:00 PM

Consider the chart below

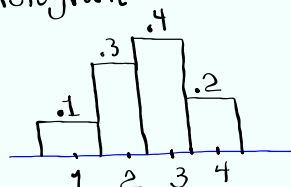
$x$	$P(x)$
1	.1
2	.3
3	.4
4	.2

1) Find  $P(x=4)$   
 $= 1 - (.1 + .3 + .4)$   
 $\uparrow$   
 Total Prob.  
 $= 1 - .8 = \boxed{.2}$

2) Find  $P(2 \leq x \leq 3)$   
 $= .3 + .4 = \boxed{.7}$

3) Draw Prob. dist. histogram

$x \rightarrow M\&$   
 $P(x) \rightarrow \text{Rel. F.}$



4) Clear all lists.

$x \rightarrow L1$ ,  $P(x) \rightarrow L2$   
 use 1-Var Stats  
 with  $L1 \& L2$

$\bar{x} = 2.7$

$S = S_x = \text{blank}$

$n = 1 \leftarrow \text{Total Prob.}$

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A piggy bank has 3 nickels & 2 dimes.

Take 2 Coins with replacement.

NN  
10¢

ND

DN

DD

15¢

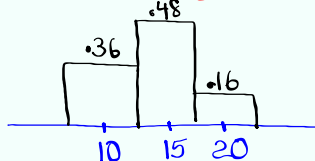
20¢

$P(10¢) = P(NN) = \frac{3}{5} \cdot \frac{3}{5} = \frac{9}{25} = \boxed{.36} \checkmark$

$P(15¢) = P(ND \text{ or } DN) = 2 \cdot \frac{3}{5} \cdot \frac{2}{5} = \frac{12}{25} = \boxed{.48} \checkmark$

$P(20¢) = P(DD) = \frac{2}{5} \cdot \frac{2}{5} = \frac{4}{25} = \boxed{.16} \checkmark$

¢	$P(¢)$
10	.36
15	.48
20	.16



¢  $\rightarrow L1$ ,  $P(¢) \rightarrow L2$

$\bar{x} = 14$

use 1-Var Stats

$S = S_x = \text{Blank}$

with  $L1 \& L2$

$n = 1 \leftarrow \text{Total Prob.}$

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working with  $x \in P(x)$ :

Mean  $\mu = \sum x p(x)$

$\mu$

Variance  $\sigma^2 = \sum x^2 p(x) - \mu^2$

$\sigma$

Standard deviation  $\sigma = \sqrt{\sigma^2}$

Oct 21-7:28 PM

Complete the chart below

$x$	$P(x)$	$xP(x)$	$x^2P(x)$	1) $\sum P(x) = $ <span style="border: 1px solid red; padding: 2px;">1</span>
2	.3	.6	1.2	2) $\sum xP(x) = $ <span style="border: 1px solid red; padding: 2px;">2.9</span>
3	.5	1.5	4.5	
4	.2	.8	3.2	
				3) $\sum x^2P(x) = $ <span style="border: 1px solid red; padding: 2px;">8.9</span>

4)  $\mu = \sum xP(x) =$  2.9

5)  $\sigma^2 = \sum x^2P(x) - \mu^2 = 8.9 - 2.9^2 =$  .49

6)  $\sigma = \sqrt{\sigma^2} = \sqrt{.49} =$  .7

68% Range  $\mu \pm \sigma = 2.9 \pm .7 \rightarrow$  2.2 to 3.6

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Using TI to find  $\mu$ ,  $\sigma$ , and  $\sigma^2$ :

$x \rightarrow L1$ ,  $P(x) \rightarrow L2$

use [1-Var Stats] with L1 & L2

$$\mu = \bar{x}$$

$$\sigma = \sigma_x$$

$$\sigma^2 = .49$$

L1	L2
2	.3
3	.5
4	.2

$$\mu = \bar{x} = 2.9$$

$$\sigma = \sigma_x = .7$$

$$n = 1$$

[VARS]  
[5: Statistics]  
[4:  $\sigma_x$ ] [ $x^2$ ]  
[Enter]

Oct 21-7:40 PM

Consider the chart below

$x$	$P(x)$
2	.3
4	.6
6	.1

Verify  $\sum P(x) = 1$

$x \rightarrow L1$ ,  $P(x) \rightarrow L2$

use [1-Var Stats]

with L1 & L2

$$\mu = \bar{x} = 3.6$$

$$\sigma = \sigma_x = 1.2$$

$$\sigma^2 = \frac{36}{25} \text{ Reduced Fraction}$$

[VARS]  
[5: Statistics]  
[4:  $\sigma_x$ ] [ $x^2$ ]  
[Math] [1:  $\rightarrow$  Frac]  
[Enter]

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A piggy bank has 3 nickels & 2 dimes.

Take 2 Coins without replacement

NN      ND      DN      DD  
10¢      15¢      20¢

$$P(10¢) = P(NN) = \frac{3}{5} \cdot \frac{2}{4} = \frac{3}{10} = \boxed{.3}$$

$$P(15¢) = P(ND \text{ or } DN) = 2 \cdot \frac{3}{5} \cdot \frac{2}{4} = \frac{6}{10} = \boxed{.6}$$

$$P(20¢) = P(DD) = \frac{2}{5} \cdot \frac{1}{4} = \frac{1}{10} = \boxed{.1}$$

¢	P(¢)
10	.3
15	.6
20	.1

$\bar{C} \rightarrow L1, P(\bar{C}) \rightarrow L2$

Use 1-Var Stats with L1 & L2

$$\mu = 14$$

$$\sigma = \sigma_x = 3$$

$$\sigma^2 = 9$$

VARS

5: Statistics

4:  $\sigma_x$   $\sigma^2$

Enter

usual Range  
"95% Range"

$$\mu \pm 2\sigma = 14 \pm 2(3) \Rightarrow \boxed{8 \text{ to } 20}$$

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Application:

$$\text{Expected Value} = \mu = \bar{x}$$

I sold 25 tkts for \$10 each.

one ticket is randomly drawn

owner gets a calc. worth \$100.

Expected Value per ticket sold.

Net    P(Net)

10-100     $\frac{1}{25}$     winning Tkt

10-0     $\frac{24}{25}$     losing tkts

net  $\rightarrow L1, P(\text{net}) \rightarrow L2$

$$E.V. = \mu = \bar{x} = \boxed{6} \quad \text{House makes } \$6/\text{TKT}$$

Use 1-Var Stats

with L1 & L2

find  $\sigma^2$

VARS 5: Statistics 4:  $\sigma_x$   $\sigma^2$  Enter

$$\sigma^2 = 384$$

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You buy insurance policy for \$100 to protect Your luggage.

Any damages, airline pays You \$1000.

Prob. of damage is .5%

Expected Value per policy Sold.

net	P(Net)
100 - 1000	.5% = .005 damage
100 - 0	.995 damage

net  $\rightarrow$  L1 use 1-Var Stats with

P(Net)  $\rightarrow$  L2 L1 & L2  
E.V. =  $\mu = \bar{x} = 95$

Find  $\sigma^2$   
= 4975

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Pay me \$5, draw a Card from a full deck of playing Cards

If you draw

Ace

Face

Any other card

I give You

\$50

\$5

\$0

net	P(Net)
5 - 50	4/52 Ace
5 - 5	12/52 Face
5 - 0	36/52 any other Card.

E.V. per bet  
for the house

$\mu = \bar{x} = 0$

$\sigma^2 = \frac{2250}{13}$

in reduced fraction

SG 14 & 15



Oct 21-8:28 PM

Binomial Prob. Dist. : (SG.16)

- 1)  $n$  independent events
- 2) Each event has only two outcomes.  
 $P(\text{Success}) = p$      $P(\text{Failure}) = q$   
 $p + q = 1$ ,  $q = 1 - p$   
 $p$  &  $q$  remain unchanged for all events.
- 3)  $X \rightarrow \#$  of Successes  
 $n - X \rightarrow \#$  of Failures

$$P(X) = {}^nC_X \cdot p^X \cdot q^{n-X}$$

# of combinations of  $X$  Successes in  $n$  events.

find  ${}^5C_2 = 10$

5 Math  $\rightarrow$  PRB  $\downarrow$   ${}^nCr$  2 enter

find  ${}^{10}C_3 = 120$

find  ${}^{50}C_5 = 2,118,760$

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Consider a binomial Prob. Dist with  
 $n = 5$  and  $p = .6$   
 $q = 1 - p = .4$

$P(\text{exactly 2 Successes})$

$$P(X = 2) = {}^5C_2 \cdot (.6)^2 \cdot (.4)^3$$

$$P(X) = {}^nC_X \cdot p^X \cdot q^{n-X}$$

$5 - 2 = 3$

$$= 10 \cdot (.6)^2 \cdot (.4)^3$$

$\approx .230$

$\wedge$   
 $\div$

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Consider a binomial Prob. dist with

$$n = 10 \quad P = .4$$

$$q = 1 - P = \boxed{.6}$$

$P(\text{exactly 6 successes})$

$$P(x=6) = {}^{10}C_6 \cdot (.4)^6 \cdot (.6)^4 = 210(.4)^6 \cdot (.6)^4$$

$$P(x) = {}^nC_x \cdot P^x \cdot q^{n-x} = \boxed{.111}$$

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we flip a fair coin 20 times.  $n=20$   
 $P=.5$   
 $q=.5$

Find the prob. that it lands exactly 12 tails.

$$P(x=12) = {}^{20}C_{12} \cdot (.5)^{12} \cdot (.5)^8$$

$$= 125970 \cdot (.5)^{12} \cdot (.5)^8$$

$$= \boxed{.120}$$

using TI:

2nd VARS ↓ ↓ ↓ binompdf(  $n, P, x$  )  
 $n \rightarrow$  Trials: 20  $P = .5$   $x = 12$  Enter

Your work

$$P(x=12) = \text{binompdf}(20, .5, 12)$$

$$= .120$$

Paste Enter

$$\boxed{.120}$$

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You are taking a quiz, multiple-choice.

$n=10$   
 $10$  questions, each question has  $4$  choices

but only  $1$  correct choice.  $P = \frac{1}{4} = .25$   
 $q = \frac{3}{4} = .75$

You are making random guesses.

$P(\text{Correctly guess on exactly } 6 \text{ questions})$

$$P(x=6) = \text{binompdf}\left(\underset{\uparrow n}{10}, \underset{\uparrow p}{.25}, \underset{\uparrow x}{6}\right) \\ = .016$$

$P(\text{Correctly guess on all questions})$

$$P(x=10) = \text{binompdf}\left(\underset{\uparrow n}{10}, \underset{\uparrow p}{.25}, \underset{\uparrow x}{10}\right) \\ = 9.54 \times 10^{-7}$$

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